

ClassTwoAlgLib

**A library of PORC functions
enumerating class two algebras over
finite fields**

Version 1.0

28 July 2017

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Acknowledgements

We appreciate very much all past and future comments, suggestions and contributions to this package and its documentation provided by **GAP** users and developers.

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Chapter 1

ClassTwoAlgLib

The package `ClassTwoAlgLib` is a small library developed together with the `ClassTwoAlg` which was designed to count the number of isomorphism classes of associative algebras of class 2 and small rank for all dimensions. See [EW17] for more information.

The package `ClassTwoAlgLib` provides the results of the enumerations of the `ClassTwoAlg` package. It is not designed to compute the functions, but it provides some functions that are also included in the `ClassTwoAlg` package.

A PORC function is in GAP designed as a multivariate polynomial over the rational numbers. The first indeterminate is displayed as q and is internally handled as indeterminate number 1000. All other indeterminates will internally have a larger number. Those indeterminates will be handled as gcd's with q . For a more detailed introduction see [EW17].

1.1 Reading the PORC functions

1.1.1 ReadNumberOfClassTwoAssociativeAlgebrasByRankAndDimension

▷ `ReadNumberOfClassTwoAssociativeAlgebrasByRankAndDimension(rk , dim)` (function)

Returns a PORC function which, when evaluated, yields the number of isomorphism types of associative algebras of rank rk and dimension dim .

Example

```
gap> ReadNumberOfClassTwoAssociativeAlgebrasByRankAndDimension(2,3);  
q-(q-0,2)+5
```

1.1.2 ReadNumberOfClassTwoAssociativeAlgebrasByRank

▷ `ReadNumberOfClassTwoAssociativeAlgebrasByRank(rk)` (function)

Returns a list of PORC functions. The i 'th function belongs to the number of isomorphism classes of nilpotent associative algebras of class 2, rank rk and dimension $rk+i$. Hence, this function calls `ReadNumberOfClassTwoAssociativeAlgebrasByRankAndDimension` (1.1.1) for every possible dimension.

Example

```
gap> ReadNumberOfClassTwoAssociativeAlgebrasByRank(2);
[ q-(q-0,2)+5, 3*q-(q-0,2)+6 ]
```

1.2 Evaluating the PORC functions

1.2.1 ValueOfPorcPolynomial

▷ ValueOfPorcPolynomial(f , x) (function)

Given a PORC function f then this function evaluates f at the point x .

Example

```
gap> f := ReadNumberOfClassTwoAssociativeAlgebrasByRankAndDimension(3,5);
q^6+q^5+3*q^4+6*q^3-2*q^2*(q-0,2)+18*q^2-7*q*(q-0,2)+q*(q-1,3)+38*q-10*(q-0,2)-1/2*(q-0,3)+89/2
gap> ValueOfPorcPolynomial(f,101);
1072348780593
gap> List([2,3,5,7,11,13], x -> ValueOfPorcPolynomial(f,x));
[ 322, 1650, 21969, 144773, 1986843, 5300147 ]
```

1.2.2 InformationOnPorcPolynomial

▷ InformationOnPorcPolynomial(f) (function)

Given a PORC function f some information of f is printed in nicely readable form. The highest degree in q can be found. Then it follows a table stating at which power of q the gcd's appear for the first time. One can stop the output when being asked "More information?" by typing 0 or *false*.

Example

```
gap> InformationOnPorcPolynomial(f);
```

```
-----
|
| SUMMARY
|
| Degree in 'q': 6
|
| Gcd's with...      2      3
| First occurrence   2      1
| (power of 'q')
|
| Total modulus of PORC function: 6.
|
| More information? (true/false)  0
```

When typing *true* or 1 a more detailed list on all gcd's is displayed.

Example

```
gap> InformationOnPorcPolynomial(f);
```

```

-----
|
| SUMMARY
|
| Degree in 'q': 6
|
| Gcd's with...      2      3
| First occurrence    2      1
| (power of 'q')
|
| Total modulus of PORC function: 6.
|
More information? (true/false)  1
|
-----
|
| INFO on 'q'
|
| Degree in 'q': 6
| Polynomial is dense in 'q'.
| (All powers (including  $q^0$ ) are present.)
|
|
-----
|
| INFO on gcds with 2
|
| (q-0,2) appears for all powers of 'q' less than or equal to 2.
|
|
-----
|
| INFO on gcds with 3
|
| (q-0,3) appears for all powers of 'q' less than or equal to 0.
| (q-1,3) first appears at power 1 of 'q'.
| Then it is present for the following power(s) of 'q' only:
| 1.
|
No further gcd's are included.
```

References

- [EW17] B. Eick and M. Wesche. classtwoalg, package to enumerate class two algebras over finite fields, Version 1.0. <http://www.icm.tu-bs.de/~morwesch/research/research.html>, 2017. GAP package. 4

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