Machine tool simulation based on reduced order FE models

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Abstract

Numerical simulations of the behavior of machine tools are usually based on a finite element (FE) discretization of their mechanical structure. After linearization one obtains a second-order system of ordinary differential equations. In order to capture all necessary details the system that inevitable arises is too complex to meet the expediency requirements of real time simulation and control. In commercial FE simulation software often modal reduction is used to obtain a model of lower order which allows for faster simulation. In recent years new methods to reduce large and sparse dynamical systems emerged. This work concentrates on the reduction of certain FE systems arising in machine tool simulation with Krylov subspace methods. The main goal of this work is to discuss whether these methods are suitable for the type of application considered here. Several Krylov subspace methods for first or second-order systems were tested. Numerical examples comparing our results to modal reduction and balanced truncation model reduction are presented.

Key words: Model Order Reduction, Simulation, Krylov Subspace, Moment Matching
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1. Introduction

The integrated simulation of machine tools consists of two major parts: the structural model of the machine tool representing its reaction on certain control inputs on the one hand and the control loop generating those inputs on the other hand. The reaction of the mechanical structure on control inputs is described by a system of FE semi-discretized partial differential equations. After linearization one obtains a system of ordinary differential equations of second order

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Fu(t), \qquad y(t) = C_v\dot{x}(t) + C_p x(t), \tag{1}$$

where $M, D, K \in \mathbb{R}^{n\times n}$, $F \in \mathbb{R}^{n\times m}$, $C_v, C_p \in \mathbb{R}^{q\times n}$, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^q$. Here Rayleigh damping is considered, that is, the damping matrix D is proportional to the mass matrix M and the stiffness matrix K: $D = \alpha \cdot M + \beta \cdot K$, where α and β are real parameters which are chosen by the experience of the design engineer and lie between 0 and 0.1. The system matrices are large, sparse and non-symmetric. All of this accounts for unacceptable computational and resource demands in simulation and control of these models. In order to reduce these demands to acceptable computational times, usually model order reduction techniques are employed which generate a reduced order model that captures the essential dynamics of the system, preserves its important properties, and has nearly the same response characteristic. Model order reduction methods are methods to find a second-order system of reduced dimension $r \ll n$

$$\tilde{M}\ddot{\tilde{x}}(t) + \tilde{D}\dot{\tilde{x}}(t) + \tilde{K}\tilde{x}(t) = \tilde{F}u(t), \qquad \tilde{y}(t) = \tilde{C}_v\dot{\tilde{x}}(t) + \tilde{C}_p\tilde{x}(t), \tag{2}$$

where $\tilde{M}, \tilde{D}, \tilde{K} \in \mathbb{R}^{r\times r}$, $\tilde{F} \in \mathbb{R}^{r\times m}$, $\tilde{C}_v, \tilde{C}_p \in \mathbb{R}^{q\times r}$, $\tilde{x}(t) \in \mathbb{R}^r$, $u(t) \in \mathbb{R}^m$, $\tilde{y}(t) \in \mathbb{R}^q$, which approximates the original system in some sense.

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Any second order model can be transformed into a first order system

$$\underbrace{\begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix}}_{E} \underbrace{\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix}}_{\dot{z}(t)} = \underbrace{\begin{bmatrix} 0 & -K \\ -K & -D \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}}_{z(t)} + \underbrace{\begin{bmatrix} 0 \\ F \end{bmatrix}}_{B} u(t), \qquad y(t) = \underbrace{\begin{bmatrix} C_p & C_v \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}}_{z(t)}, \tag{3}$$

where $E, A \in \mathbb{R}^{2n\times 2n}$, $B \in \mathbb{R}^{2n\times m}$, $C \in \mathbb{R}^{q\times 2n}$, $z(t) \in \mathbb{R}^{2n}$, $u(t) \in \mathbb{R}^m$, $y(t) \in \mathbb{R}^q$. Various other linearizations have been proposed in the literature, see, e.g., [1]. The linearization (3) is usually preferred as it is symmetry preserving in case K, M, D are symmetric. The system considered here is non-symmetric, so one of the various other possible linearizations could be used instead. Note that by the transformation process the dimension of the system doubles. The corresponding reduced order system is of the form

$$\tilde{E}\dot{\tilde{z}}(t) = \tilde{A}\tilde{z}(t) + \tilde{B}u(t), \qquad \tilde{y}(t) = \tilde{C}\tilde{z}(t), \tag{4}$$

where $\tilde{E}, \tilde{A} \in \mathbb{R}^{r\times r}$, $\tilde{B} \in \mathbb{R}^{r\times m}$, $\tilde{C} \in \mathbb{R}^{q\times r}$, $\tilde{z}(t) \in \mathbb{R}^r$, $u(t) \in \mathbb{R}^m$, $\tilde{y}(t) \in \mathbb{R}^q$.

In engineering modal reduction [2] is most common. This method has the disadvantage that the reduced system only contains information of the modes chosen to generate the reduced system. Moreover, the choice of the essential modes is usually based on a heuristic knowledge of the design engineer and cannot be fully automated. Therefore, there is a need for alternative reduction methods which can be fully automated. In the last years new reduction methods to reduce large and sparse dynamical systems were presented, see [3] for an overview of methods for linear systems. The two most famous methods are balanced truncation approximation (BTA) and Krylov subspace methods. Here we consider the reduction of large structural mechanical FE models by Krylov subspace methods.

2. Model reduction via Krylov subspace methods

The Laplace transform of the impulse response H (that is, the transfer function) is, for the systems considered here, a rational function. One way to approximate a system is to approximate its transfer function by a rational function of lower degree. This can be done by matching some terms of the Laurent series expansion of H at various points of the complex plane. This problem can be solved in a numerically efficient way by employing Krylov subspace projection based model reduction methods. Such methods generate a Petrov-Galerkin projection $\Pi = VW^T$ which can be used to project the large system (1) (resp., (3)) onto a small system (2) (resp. (4)) of dimension $r \ll n$ (resp. $r \ll 2n$).

The reduced second-order system (2) is constructed by applying the Petrov-Galerkin projection to (1) such that

$$\tilde{M} = W^T M V, \quad \tilde{D} = W^T D V, \quad \tilde{K} = W^T K V, \quad \tilde{F} = W^T F, \quad \tilde{C}_p = C_p V, \quad \text{and} \quad \tilde{C}_v = C_v V, \tag{5}$$

where $V, W \in \mathbb{R}^{n\times r}$ with $W^T V = I_r$. The reduced first-order system (4) is constructed by applying the Petrov-Galerkin projection $\Pi = VW^T$ to (3) such that

$$\tilde{E} = W^T E V, \quad \tilde{A} = W^T A V, \quad \tilde{B} = W^T B, \quad \text{and} \quad \tilde{C} = CV, \tag{6}$$

where $V, W \in \mathbb{R}^{2n\times r}$ with $W^T V = I_r$, or with $V^T V = I_r$ and $W^T W = I_r$. The matrices V and W will be build from suitable bases of certain (block) Krylov subspaces. A kth order (block) Krylov subspace is defined by

$$\mathcal{K}_k(P, Q) = \operatorname{span}\{Q, PQ, P^2Q, \ldots, P^{k-1}Q\} \tag{7}$$

where $P \in \mathbb{R}^{n\times n}$ and $Q \in \mathbb{R}^{n\times \ell}$. In case, $\ell = 1$ we consider a standard Krylov subspace with the starting vector $Q = q$. Otherwise, the columns of $Q = [q_1, \ldots, q_\ell] \in \mathbb{R}^{n\times \ell}$ should be linearly independent. In case a Galerkin projection ($V = W$) is sought, V results from orthogonalizing a single (block) Krylov sequence. In case a Petrov-Galerkin projection is sought, V and W ($V \neq W$) are essentially constructed from (bi-)orthogonalizing two (block) Krylov sequences. Usually, this is achieved by employing either the (block) Arnoldi or the (block) Lanczos algorithm. For more details on Krylov subspace methods see, e.g. [4, 5].

2.1. First order systems

Expanding the transfer function of (3) in a Laurent expansion series around an expansion point s_0 yields the moments $h_j(s_0)$, $j = 0, \ldots, \infty$ of the transfer function

$$H(s) = C(sE - A)^{-1}B = \sum_{j=0}^{\infty} h_j(s_0)(s - s_0)^j,$$

where

$$h_j(s_0) = -C[(A - s_0E)^{-1}E]^j(A - s_0E)^{-1}B.$$

Consider the block Krylov subspace (7) for $P^{(1)} = (A - s_0E)^{-1}E$ and $Q^{(1)} = -(A - s_0E)^{-1}B$. Assume that an orthogonal basis for this block Krylov subspace is generated using a suitable method (e.g., the block Arnoldi method). Choose the column vectors of $V = W$ as the first r vectors of that basis and apply the congruence transformation (6) with V, $W = V$ and $V^TV = I_r$. Then the transfer function of the resulting reduced system matches at least the first $\lfloor r/m \rfloor$ moments of the transfer function of the original system [3].

That is, at least the first $\lfloor r/m \rfloor$ moments $\tilde{h}_j(s_0)$, of the transfer function $\tilde{H}(s)$ of the reduced system (4) equal the first moments $h_j(s_0)$, of the transfer function $H(s)$ of the original system (3) at the expansion point s_0

$$\tilde{h}_j(s_0) = h_j(s_0), \qquad j = 0, 1, \ldots, \lfloor r/m \rfloor - 1.$$

When V is determined from the block Krylov subspace generated by $P^{(1)}$ and $Q^{(1)}$, while W is determined from the block Krylov subspace generated by $P^{(2)} = (A - s_0E)^{-T}E^T$ and $Q^{(2)} = -(A - s_0E)^{-T}C^T$, such that $W^TV = I_r$ (e.g., by a block two-sided Lanczos method) or $V^TV = I_r$ and $W^TW = I_r$ (e.g., by a block two-sided Arnoldi method), then the transfer function of the system obtained by applying the congruence transformation (6) with V and W matches at least the first $\lfloor r/m \rfloor + \lfloor r/q \rfloor$ moments of the transfer function of the original system [3].

As the reduced order models obtained in this fashion depend heavily on the chosen expansion point s_0, an alternative is to use more than one expansion point. Such methods are called Rational-Krylov methods [6]. Assume that $\hat{\imath}$ expansion points s_i, $i = 1, 2, \ldots, \hat{\imath}$ are to be considered. Let $r_i \in \mathbb{N}$ and $\sum_{i=1}^{\hat{\imath}} r_i = r$. Now, these methods require that at least the first $\lfloor r_i/m \rfloor$ moments $\hat{h}_j(s_i)$ of the transfer function $\hat{H}(s)$ of the reduced system equal the first moments $h_j(s_i)$ of the transfer function of the original system $H(s)$ at the expansion points s_i, $i = 1, 2, \ldots, \hat{\imath}$:

$$\tilde{h}_j(s_i) = h_j(s_i), \qquad j = 0, 1, \ldots, \lfloor r_i/m \rfloor - 1, \quad i = 1, 2, \ldots, \hat{\imath},$$

where, as before, $\tilde{h}_j(s_i)$ are the moments of the reduced system transfer function $\tilde{H}(s)$. Assume that an orthogonal basis for each of the block Krylov subspaces $\mathcal{K}(P_i^{(3)}, Q_i^{(3)})$ generated by $P_i^{(3)} = (A - s_iE)^{-1}E$ and $Q_i^{(3)} = -(A - s_iE)^{-1}B$, $i = 1, 2, \ldots, \hat{\imath}$ is computed using a suitable method (e.g., the block Arnoldi method). Then, $V = W$ is build by using the first r_i bases vectors of each of these Krylov subspaces. The transfer function of the resulting reduced system matches at least the first $\lfloor r_i/m \rfloor$ moments of the transfer function of the original system at the expansion points s_i, $i = 1, 2, \ldots, \hat{\imath}$.

When V is determined from the block Krylov subspaces generated by $P^{(3)}$ and $Q^{(3)}$, while W is determined from the block Krylov subspaces generated by $P_i^{(4)} = (A - s_iE)^{-T}E^T$ and $Q_i^{(4)} = -(A - s_iE)^{-T}C^T$, such that $W^TV = I_r$, resp. $V^TV = I_r$ and $W^TW = I_r$, then the transfer function of the system obtained by applying the congruence transformation (6) with V and W matches at least the first $\lfloor r_i/m \rfloor + \lfloor r_i/q \rfloor$ moments of the transfer function of the original system at the expansion points s_i, $i = 1, 2, \ldots, \hat{\imath}$ [3].

Without prior knowledge of the systems eigenvalues, it is difficult to choose the initial set of expansion points in a (near) optimal way. Therefore, in [7, 8] an iterative procedure for the choice of expansion points $s_i, i = 1, \ldots, \hat{\imath}$ is discussed. Starting from an initial set of expansion points s_i a reduced order system is determined. Then a new set of expansion points is chosen as $s_i = -\lambda_i, i = 1, \ldots, \hat{\imath}$ where λ_i are the eigenvalues of the matrix pencil $\tilde{E} - \lambda\tilde{A}$ with $\tilde{E}, \tilde{A}$ as in (4). This algorithm is called Iterative Rational-Krylov Algorithm (IRKA). Here a modified version of IRKA is proposed: A new set of expansion points is chosen from the set of eigenvalues ordered by their imaginary part such that $|\mathrm{Im}(\lambda_1)| \leq |\mathrm{Im}(\lambda_2)| \leq \ldots \leq |\mathrm{Im}(\lambda_r)|$. Starting from $s_1 = \mathrm{Im}(\lambda_1) \cdot \imath$ ($\imath = \sqrt{-1}$) the next expansion points $s_i, i = 2, \ldots, \hat{\imath}$ are chosen as $s_i = \mathrm{Im}(\lambda_i) \cdot \imath$. As expansion points lying a bit apart yield better approximation results, this choice of the expansion points is refined such that in addition we require $|s_{i-1} - s_i| > \epsilon$, where ϵ is chosen by the user

and defines a (minimum) distance between two adjacent expansion points. Hence, if $|s_2 - s_1| \leq \epsilon$, we do not choose $s_2 = \mathrm{Im}(\lambda_2) \cdot \imath$, but test $|s_2 - s_1|$ for $s_2 = \mathrm{Im}(\lambda_3) \cdot \imath$. If this is still less than ϵ, we next test for $s_2 = \mathrm{Im}(\lambda_4) \cdot \imath$, until we have found an λ_k such that $s_2 = \mathrm{Im}(\lambda_k) \cdot \imath$ yields $|s_2 - s_1| > \epsilon$. Next we choose s_3 in the same fashion starting from λ_{k+1} such that $|s_3 - s_2| > \epsilon$. Unlike IRKA, this method cannot be guaranteed to be $\mathcal{H}_2$-optimal. After a few iterations good approximation results of the transfer function especially for low frequencies are obtained. The approach described here is called Modified Iterative Rational Arnoldi Algorithm (MIRKA).

2.2. Second order systems

Expanding the transfer function of (1) in a Laurent expansion series around an expansion point s_0 yields the moments $h_j(s_0)$, $j = 0, \ldots, \infty$ of the transfer function

$$H(s) = (C_p + sC_v)(s^2M + sD + K)^{-1}F = \sum_{j=0}^{\infty} h_j(s_0)(s - s_0)^j,$$

where $h_0(s_0) = \hat{C}_p\zeta_0$ and

$$h_j(s_0) = C_v\zeta_{j-1} + \hat{C}_p\zeta_j, \qquad j = 1, 2, \ldots$$

with $\hat{D} = 2s_0M + D$, $\hat{K} = s_0^2M + s_0D + K$ and $\hat{C}_p = C_p + s_0C_v$ and

$$\begin{aligned} \zeta_0 &= \hat{K}^{-1}F \\ \zeta_1 &= \hat{K}^{-1}(-\hat{D}\zeta_0) \\ \zeta_j &= \hat{K}^{-1}(-\hat{D}\zeta_{j-1} - M\zeta_{j-2}), \quad j = 2, 3, \ldots. \end{aligned}$$

Moment matching can be achieved similar to the approach for first order systems. As methods using more than one expansion point are usually more effective, only those are considered here.

When more than one expansion point is used the column vectors of matrix V are determined from the block Krylov subspaces $P_i^{(5)} = -(s_i^2M + s_iD + K)^{-1}M$ and $Q_i^{(5)} = (s_i^2M + s_iD + K)^{-1}F$, while the column vectors of matrix W are determined from the block Krylov subspaces $P_i^{(6)} = -(s_i^2M + s_iD + K)^{-T}M^T$ and $Q_i^{(6)} = (s_i^2M + s_iD + K)^{-T}(C_p + s_iC_v)^T$. The transfer function of the system obtained by applying the congruence transformation (5) with V and W matches at least the first $\lfloor r_i/m \rfloor + \lfloor r_i/q \rfloor$ moments of the original system transfer function at the expansion points $s_i, i = 1, \ldots, \hat{i}$ [9]. As in the case of first-order systems, an iterative approach for the choice of the expansion points s_i can be used.

3. Numerical Results

Our test model is a simplified, abstract mechanical structure of a machine tool designed using the CAD environment NASTRAN© (see Figure 1 with courtesy of iwb[1], here TCP denotes the tool center point). The test model is of

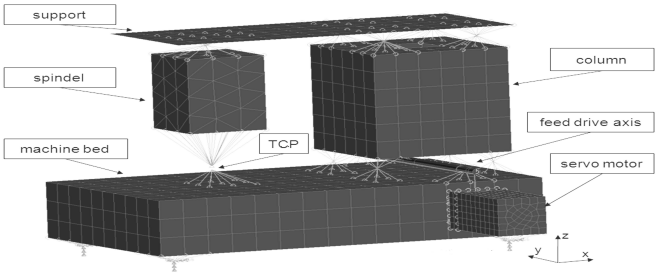

Figure 1: FE model of a simplified, abstract mechanical structure.

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order $n = 4983$, it has four inputs ($m = 4$) and eight outputs. Due to the modeling, the output vectors do not span an 8-dimensional space. Here only the first four linear independent output vectors, which was the 1st, 4th, 5th and 6th vector, were used so that $m = q = 4$. The parameters for the proportional damping matrix were chosen as $\alpha = 0.02$ and $\beta = \alpha/100$.

To test the approximation abilities of MIRKA various methods to determine the matrices V and W were used. To reduce first oder systems the following Krylov subspace methods were used:

1. Arnoldi method (AM) [10] which generates a Petrov-Galerkin projection $\Pi = WV^T, V^TV = I_r, W^TW = I_r$ from $\mathcal{K}(P^{(1)}, Q^{(1)})$ and $\mathcal{K}(P^{(2)}, Q^{(2)})$ by a block two-sided Arnoldi method.
2. Lanczos method (LM) [11] which generates a Petrov-Galerkin projection $\Pi = WV^T, W^TV = I_r$ from $\mathcal{K}(P^{(1)}, Q^{(1)})$ and $\mathcal{K}(P^{(2)}, Q^{(2)})$ by a block two-sided Lanczos method.
3. Rational-Arnoldi method (RA) [6] which generates a Petrov-Galerkin projection $\Pi = WV^T, V^TV = I_r, W^TW = I_r$ from $\mathcal{K}(P^{(3)}, Q^{(3)})$ and $\mathcal{K}(P^{(4)}, Q^{(4)})$ by a block two-sided Arnoldi method.
4. Rational-Lanczos method (RL) [12] which generates a Petrov-Galerkin projection $\Pi = WV^T, W^TV = I_r$ from $\mathcal{K}(P^{(3)}, Q^{(3)})$ and $\mathcal{K}(P^{(4)}, Q^{(4)})$ by a block two-sided Lanczos method.

The first two methods use only one expansion point, while the last two methods employ several expansion points chosen by the procedure described above. In case complex valued expansion points are used, all of the above algorithms generate complex valued matrices V and W.

To reduce the second-order system the following Krylov subspace methods were used:

1. Rational Arnoldi for systems without damping matrix (RA_2o) [6, 13] which generates a Petrov-Galerkin projection $\Pi = WV^T, V^TV = I_r, W^TW = I_r$ from $\mathcal{K}(P^{(5)}, Q^{(5)})$ and $\mathcal{K}(P^{(6)}, Q^{(6)})$ by a block two-sided Arnoldi method.
2. Rational Lanczos for systems without damping matrix (RL_2o) [12, 13] which generates a Petrov-Galerkin projection $\Pi = VW^T, W^TV = I_r$ from $\mathcal{K}(P^{(5)}, Q^{(5)})$ and $\mathcal{K}(P^{(6)}, Q^{(6)})$ with $D = 0$ by a block two-sided Lanczos method.
3. Rational Arnoldi for systems with proportional damping (RA_PD) [9] which generates a Petrov-Galerkin projection $\Pi = WV^T, V^TV = I_r, W^TW = I_r$ from $\mathcal{K}(P^{(5)}, Q^{(5)})$ and $\mathcal{K}(P^{(6)}, Q^{(6)})$ by a block two-sided Arnoldi method.
4. Rational Lanczos for systems with proportional damping (RL_PD) which generates a Petrov-Galerkin projection $\Pi = VW^T, W^TV = I_r$ from $\mathcal{K}(P^{(5)}, Q^{(5)})$ and $\mathcal{K}(P^{(6)}, Q^{(6)})$ by a block two-sided Lanczos method.

These methods use more than one expansion point chosen by the procedure described above. The first two methods are modified versions of the Rational-Arnoldi resp. the Rational-Lanczos method for first-order systems. They reduce second-order systems without damping matrix. That is, they assume $D = 0$ in (1) and compute a reduced system (2) with $\tilde{D} = 0$. A damped reduced system is obtained by adding the proportional damping matrix $\tilde{D} = \alpha\tilde{M} + \beta\tilde{K}$. The last two methods exploit the special structure of the proportional damping matrix. In case complex valued expansion points are used, only the last two of the above algorithms generate complex valued matrices V and W. The algorithms RA_2o and RL_2o generate real matrices even in case of complex valued expansion points.

All implementations exploit the sparsity of the system matrices. The pseudo code of MIRKA is given as Algorithm 1. For the pseudo code of the several Krylov subspace methods see [14]. The algorithms were implemented in MATLAB[2] version 7.1 (R14). The computations were performed on a AMD Athlon(tm) 64 X2 Dual Core Processor 4400+ and 2 GB RAM.

The focus of the numerical experiments in this paper is on generating reduced systems of a certain, user chosen dimension using a user chosen number of expansion points.

As the reduced systems are used for further simulation in NASTRAN or SIMULINK which requires real first oder systems, the following considerations had to be taken into account. The user chosen dimension r specifies the dimension of the reduced first-order system. Therefore as the dimension of second-order models doubles by transformation into a first-order system the dimension of the reduced second-order system is $r/2$. Once a complex expansion point is used, in most of the algorithms all further computations involve complex arithmetic. Hence,

[2]MATLAB is a trademark of The MathWorks, Inc.

Algorithm 1 Modified Iterative Rational-Krylov Algorithm (MIRKA)

Input: system matrices, initial expansion points s_i, $i = 1, \ldots, \hat{i}$,
reduced dimension r as well as r_i, $i = 1, \ldots, \hat{i}$, tolerance tol, ϵ
Output: reduced system of the order r

1: **while** $\max_{i \in \{1,2,\ldots \hat{i}\}} |s_i^{old} - s_i| < tol$ **do**
2: $V = [\,]$ and $W = [\,]$.
3: compute V and W using a Krylov subspace method described above
4: compute reduced system matrices with V and W by (5) resp. (6)
5: compute the eigenvalues λ_j, $j = 1 \ldots, r$ of the reduced system ordered such that $|\mathrm{Im}(\lambda_1)| \leq |\mathrm{Im}(\lambda_2)| \leq \ldots \leq |\mathrm{Im}(\lambda_r)|$
6: $s_i^{old} \leftarrow s_i$, for $i = 1, \ldots, \hat{i}$
7: choose new expansion points s_i as explained at the end of Section 2.1
8: **end while**
9: compute the congruence transformation with V and W by (5) resp. (6)

before computing the congruence transformation in the final step 9 the transformation matrices V and W have to be transformed back to real matrices. This can be done, e.g., as follows

$$[V, R] = \mathrm{qr}(\,[\mathrm{Re}(V(:, 1 : \lceil s/2 \rceil))\ \mathrm{Im}(V(:, 1 : \lfloor s/2 \rfloor))]\,), W = V,$$

resp.

$$V = (\,[\mathrm{Re}(V(:, 1 : \lceil s/2 \rceil))\ \mathrm{Im}(V(:, 1 : \lfloor s/2 \rfloor))]\,), W = (\,[\mathrm{Re}(W(:, 1 : \lceil s/2 \rceil))\ \mathrm{Im}(W(:, 1 : \lfloor s/2 \rfloor))]\,),$$

where $s = r$ for first-order systems and $s = r/2$ for second-order systems. By this transformation process the number of columns of V and W doubles. Therefore, in setting up the real transformation matrices only the first $s/2$ columns of V (and W) are used so that the resulting system is of the desired order s. Note that by this process the number of matched moments may decreases in some expansion points.

In the experiments reported here, the methods AM and LM were used with the single initial expansion point $100\pi\imath$. All other methods results were started with the four initial expansion points $2\pi\imath$, $500\pi\imath$, $1000\pi\imath$ and $1500\pi\imath$. First order systems were reduced to an order of 50, while second-order systems were reduced to an order of 25. In our computations, $\hat{i} = 4$, $tol = 0.1$ and $\epsilon = 1000$ was used.

In our current implementation the dimension of the Krylov subspaces generated was chosen as $\ell_i = k_i \cdot m$ where $k_i = \lceil r/(2\hat{i}m) \rceil$ (taking $m = q$ into account). This may lead to the computation of more vectors than needed. E.g., for $r = 50$, $m = \hat{i} = 4$ and $k_i = 2$ we have computed 32 vectors instead of just 25. Hence 7 vectors have to be omitted. This is done by omitting the final 4 vectors in the basis of the Krylov subspaces corresponding to s_4 and omitting the final 3 vectors in the basis of the Krylov subspaces corresponding to s_3. In other words, the matrices V and W are build from the basis vectors of the Krylov subspaces by first using first m vectors of all subspaces, than the second m vectors, and so on until all vectors have been used. Finally, the last $\hat{i}\ell_i - s$ vectors are deleted. This way, at each expansion point Krylov subspaces of at least dimension $(k_i - 1) \cdot m$ are used, for some expansion points Krylov subspaces of dimension $k_i \cdot m$ are used, so that in our context here at least $2(k_i - 1)$ moments are matched at each expansion point.

For the methods RA_PD and RL_PD we choose to first compute V and $W \in \mathbb{C}^{13 \times 13}$. This implies that for each expansion point only a subspace of dimension 4 can be used, so that at first for each expansion point 2 moments are matched. But as $4 \cdot 4 = 16$ vectors are still more than needed, 3 vectors have to be deleted. As described above, this is done by omitting the final 3 vectors of the basis of the Krylov subspaces corresponding to s_4. Therefore, no moment is matched for the last expansion point, only the first vector of the corresponding Krylov subspaces are used in V and W. When transforming these matrices to real transformation matrices by the above described procedure, the resulting matrices will be of dimension 25×25, that is, from the only vector from the Krylov subspaces corresponding to s_4 used in setting up V and W only the real part is used in the final transformation.

In Table 1 essential information about the results obtained and information used is summarized. All methods converged in at most 4 iteration steps. Except for the method RA the expansion point(s) converged to the same (set of) expansion point(s) in all methods. Besides the methods already mentioned, a second order modal reduced system

generated by NASTRAN of dimension 25 (modal_2o) was computed in order to compare the approximation results of various reduction methods.

first-order methods							
	$V, W \in$	number of vectors r_i per expansion point s_i	moments matched per expansion point s_i	expansion points in the last iteration s_0 resp. s_i	k_i	time to reduce the system [s]	number of iterations
AM	$\mathbb{C}^{25\times25}$	25	12	$2\pi \cdot 75.68\imath$	7	47.97	2
LM	$\mathbb{C}^{25\times25}$	25	12	$2\pi \cdot 75.68\imath$	7	50.94	2
RA	$\mathbb{C}^{25\times25}$	8, 8, 5, 4	4, 4, 2, 2	$2\pi(75.68\imath, 253.66\imath, 495.95\imath, 685.75\imath)^T$	2	119.57	3
RL	$\mathbb{C}^{25\times25}$	8, 8, 5, 4	4, 4, 2, 2	$2\pi(75.68\imath, 253.68\imath, 430.41\imath, 633.11\imath)^T$	2	121.11	3

second-order methods							
	$V, W \in$	number of vectors r_i per expansion point s_i	moments matched per expansion point s_i	expansion points in the last iteration s_i	k_i	time to reduce the system [s]	number of iterations
RA_2o	$\mathbb{R}^{25\times25}$	8, 8, 5, 4	4, 4, 2, 2	$2\pi(75.68\imath, 253.68\imath, 430.41\imath, 633.11\imath)^T$	2	37.23	4
RL_2o	$\mathbb{R}^{25\times25}$	8, 8, 5, 4	4, 4, 2, 2	$2\pi(75.68\imath, 253.68\imath, 430.41\imath, 633.11\imath)^T$	2	64.80	4
RA_PD	$\mathbb{C}^{13\times13}$	4, 4, 4, 1	2, 2, 2, 0	$2\pi(75.68\imath, 253.68\imath, 430.41\imath, 633.18\imath)^T$	2	55.89	4
RL_PD	$\mathbb{C}^{13\times13}$	4, 4, 4, 1	2, 2, 2, 0	$2\pi(75.68\imath, 253.68\imath, 430.41\imath, 633.11\imath)^T$	2	29.95	2

Table 1: Some results for different first and second order methods

In order to compare the different methods the approximation of the original transfer function and the time response of the reduced systems were analyzed. To assess the quality of the reduced systems the following errors were used:

- The absolute time response error from the jth input to the kth output of a reduced system was computed by

$$\epsilon_{t,abs}(t) = |y_{j,k}(t) - \tilde{y}_{j,k}(t)|.$$

  Here $\epsilon_{t,abs}(t)$ is the absolute error, $y_{j,k}(t)$ is the kth output of the original system and $\tilde{y}_{j,k}(t)$ is the kth output of the reduced system at time t.

- The relative error of the transfer function from the jth input to the kth output of a reduced system was computed by

$$\epsilon_{\omega,rel}(\omega) = \frac{|H_{j,k}(2\pi\,\omega\,\imath) - \tilde{H}_{j,k}(2\pi\,\omega\,\imath)|}{|H_{j,k}(2\pi\,\omega\,\imath)|}.$$

  Here $\epsilon_{\omega,rel}(\omega)$ is the relative error, $H_{j,k}$ and $\tilde{H}_{j,k}$ are the transfer functions from the jth input to the kth output of the original resp. of the reduced system.

3.1. Approximation of the transfer function

In Figure 2 the relative approximation errors of the transfer function from the 2nd input to the 4th output of the original system are given. All figures show the relevant frequency interval, which for our application is the one from 0 to 750 Hz.

The results for the first-order systems are displayed on the left in each of the figures, while the results for the second-order systems are given on the right hand side. Besides the different MIRKA-reduced models all figures also include the second-order modal reduced system (modal_2o) of dimension 25 as generated in NASTRAN, which, for first-order systems was transformed into a first-order system of dimension 50 by (3) (modal_1o).

Clearly, the Krylov subspace reduced first-order systems approximate the original system more accurate than the modal reduced system in the frequency interval considered here. All four methods achieve reduced systems with lower approximation errors than the modal reduced system of the same order. The approximation results of the reduced systems AM and LM are similar to each other and of high accuracy up to a frequency of 330 Hz. The higher the frequency the lower the accuracy of these two methods. This is mainly due to the departure from the final expansion point used, which was located at 75.68 Hz. The reduction with Rational-Krylov methods which use more than one expansion point yields better approximations than AM and LM for frequencies higher than 430 Hz.

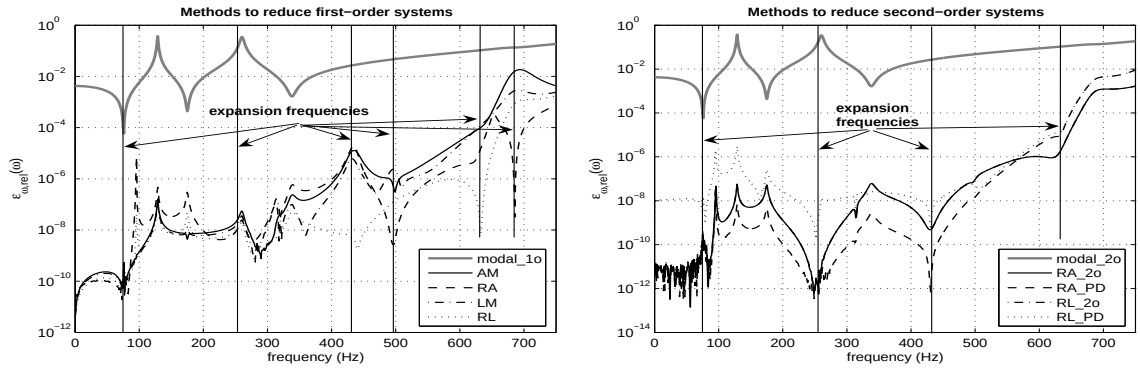

Figure 2: Relative error $\epsilon_{\omega,rel}(\omega)$ of the reduced systems.

For the reduction of the second-order system only methods which use four expansion points were considered. Their behavior is quite similar to each other and to the result obtained by the methods RA and RL. All four methods achieve reduced systems with lower approximation errors than the modal reduced system. They yield good approximations in a wide range of frequencies, especially close to the expansion points. The methods RA_2o and RL_2o have nearly the same errors. The reduced system RA_PD has slightly lower errors as the other methods in the frequency range 100 to 560 Hz. Hence, for the application considered here, the method RA_PD should be preferred to reduce systems with proportional damping matrix. This method is more accurate with nearly the same computational effort as the method RL_PD.

3.2. Approximation of the time response

To analyze the approximation abilities of the response behavior in time the reduced models were embedded into a control loop designed with MATLAB/SIMULINK, see [14] for some details. The control loop was designed by the iwb, they also produced Figure 3. In Figure 3 the absolute errors of the time responses after embedding the reduced systems into the control loop are given. Due its size, the full system could not be used in order to generate a reference solution. As a reference solution the time response of a modal reduced system of order 400 was used. Clearly, the Krylov subspace methods yield good approximations of the original system time response. In the time

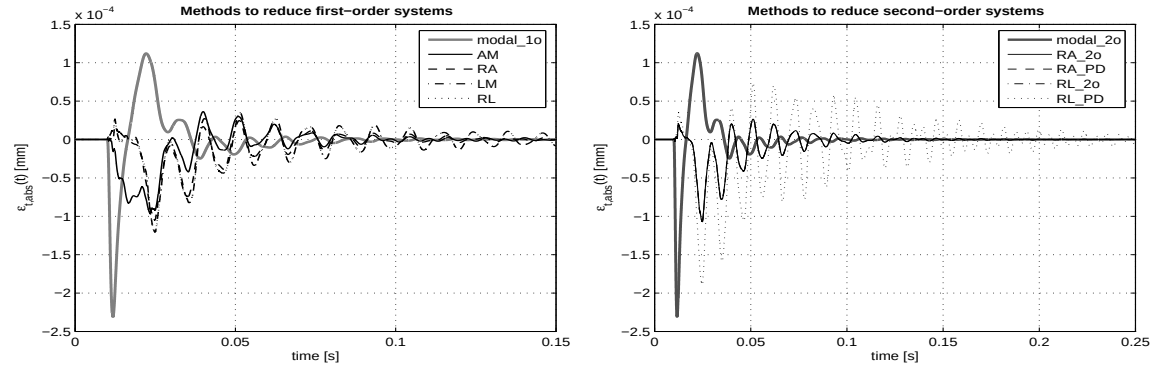

Figure 3: Absolute error $\epsilon_{t,abs}(t)$ of the reduced systems.

interval between 0 and 0.03 s, all reduced systems obtained by Krylov subspace methods, except RL_PD, approximate the time response of the original system more exactly than the modal reduced system of the same order. In the rest of the time interval the Krylov subspace reduced systems results in time responses which oscillate slightly more than the time response of the modal reduced system. The reduced system obtained by RL_PD approximates the original time response less accurate than all other reduced systems obtained by methods to reduce second order systems.

4. Conclusions

We propose a modified reduction method based on IRKA adapted for the special properties of reduced models occurring in the simulation of machine tools, called MIRKA. Several Krylov methods to reduce first- or second-order systems were implemented to test the method.

For the problem at hand the methods with more than one expansion point should be used, they give better results at a wider range of frequencies. In time response we obtained similar approximation results whether by using reduction methods for first-order systems or second-order systems. For the entire frequency range considered here the reduced systems obtained with second order methods have a better relative error than the reduced systems obtained with first order methods. So the methods for reducing second-order systems are to prefer for problems with very large system matrices.

Krylov subspace methods yield reduced models approximating the original transfer function and time response quite well. Krylov subspace methods are suitable for the reduction of structural mechanical FE models obtained by CAD environments like NASTRAN. With these methods reduced models are obtained in a very effective way which have better approximation abilities like modal reduced ones.

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